DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD -402 103

Semester Winter Examination – Dec.- 2019

Branch: B. Tech. (Common to all) **Semester:- I**
Subject:- Engineering Mathematics – I (MATH 101) **Marks: 60**
Date:- 11/12/2019 **Time:- 3 Hr.**

Instructions to the Students

1. Attempt **any five** questions of the following.
2. Illustrate your answers with neat sketches, diagram etc., wherever necessary.
3. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly

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Q.1

(a) Determine the consistency of the set of equations:

$x - 2y + z = -5\ ;\ x + 5y - 7z = 2\ ;\ 3x + y - 5z = 1$. **[6 *Marks*]**

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **[6 *Marks*]**

Q.2

(a) If $y = x^n \log x$, prove that $y_{n+1} = \frac{n!}{x}$. **[6 *Marks*]**

(b) Using Taylor's theorem,

Prove that $\log \sin x = \log \sin a + (x-a)\cot a - \frac{1}{2}(x-a)^2 \operatorname{cosec}^2 x + \cdots$ **[6 *Marks*]**

Q.3 Solve any TWO:

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$. **[6 *Marks*]**

(b) If z is a homogeneous function of degree n in x *and* y , prove that

$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z.$ **[6 *Marks*]**

(c) If $z = f(x, y)$ where $x = e^u + e^{-v}$ & $y = e^{-u} - e^v$,

then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. **[6 *Marks*]**

Q.4

(a) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. **[4 *Marks*]**

(b) Find the percentage error in the measurement of the area of an ellipse when an error of **1.5 %** is made in measuring its major and minor axes. [4 *Marks*]

(c) Find the points on the surface $z^2 = xy + 1$ nearest to the origin. [4 *Marks*]

Q.5 Solve any TWO:

(a) Evaluate the integral $I = \int_0^1 \int_0^x e^{x+y}\, dy\, dx$. [6 *Marks*]

(b) Change the order of integration and evaluate $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\cos y}{y}\, dx\, dy.$ [6 *Marks*]

(c) Evaluate the integral $I = \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dx\, dy$. [6 *Marks*]

Q.6

(a) State D' Alembert's ratio test, and hence check the convergence of the series:
$\sum_{n=1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2}\right)$. [6 *Marks*]

(b) State Cauchy's root test, and hence check the convergence of the series:
$\sum \frac{[(2n+1)\, x]^n}{n^{n+1}}$ $(x > 0)$. [6 *Marks*]

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