DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD
Semester Winter Examination – December - 2019

Branch: B. Tech. (Common to all) **Semester: II**

Subject with Subject Code: Engineering Mathematics – II (BTMA 201) **Marks: 60**

Date: 09.12.2019 **Time: 3 Hrs.**

Instructions to the Students

1. Attempt **any five** questions of the following.
2. Illustrate your answers with neat sketches, diagrams, etc., wherever necessary.
3. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

Q. 1

(a) If the sum and product of two complex numbers are real, show that those two numbers must be either real or conjugate. [4 *Marks*]

(b) Solve the equation $x^6 - i = 0$. [4 *Marks*]

(c) If $\tan(A + iB) = x + iy$, prove that

(i) $\tan 2A = \frac{2x}{1-x^2-y^2}$ (ii) $\tanh 2B = \frac{2y}{1+x^2+y^2}$. [4 *Marks*]

Q. 2

(a) Solve: $\cos^2 x \frac{dy}{dx} + y = \tan x$. [4 *Marks*]

(b) Solve: $(x^2 + y^2)dx - xy\, dy = 0$. [4 *Marks*]

(c) A body falling from rest is subjected to the force of gravity and an air resistance of $\left(\frac{n^2}{g}\right)$ times square of the velocity. Show that the distance travelled by the body in t seconds is $\frac{g}{n^2} \log \cos h\,(nt)$. [4 *Marks*]

Q. 3 Solve any THREE:

(a) Solve $(D^6 - D^4)y = x^2$. [4 *Marks*]

(b) Solve $(D^2 - 2D + 1)y = x\, e^x \cos x$. [4 *Marks*]

(c) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$. [4 *Marks*]

(d) Solve: $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$. [4 *Marks*]

Q. 4 Solve any TWO:

(a) Find the Fourier series of the function $\boldsymbol{f(x) = x}$ in the interval $(\mathbf{0, 2\pi})$. [6 *Marks*]

(b) Find the Fourier series expansion for the function $\boldsymbol{f(x) = x - x^2}$ in $\mathbf{-1 < x < 1}$. [6 *Marks*]

(c) Expand the function $f(x) = \pi x - x^2$ in a half-range sine series in the interval $(0, \pi)$. [6 *Marks*]

Q. 5 Solve any THREE

(a) Find the value of the constant λ such that the vector field defined by

$\vec{F} = (2x^2y^2 + z^2)\hat{\imath} + (3xy^3 - x^2z)\hat{\jmath} + (\lambda xy^2z + xy)\hat{k}$ is solenoidal. [4 *Marks*]

(b) Find $\nabla \cdot \vec{F}$, where $\vec{F} = \left(\frac{x}{r}\right)\hat{\imath} + \left(\frac{y}{r}\right)\hat{\jmath} + \left(\frac{z}{r}\right)\hat{k}$. [4 *Marks*]

(c) Find **curl** $\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. [4 *Marks*]

(d) If $\vec{r}$ is a position vector with $r = |\vec{r}|$, show that

$\nabla^2 r^n = n(n+1)r^{n-2}$. [4 *Marks*]

Q. 6:

(a) Find the values of the line integral $\int_C \vec{F} \cdot d\vec{r}$ along the path

$y^2 = x$ joining the points $(0, 0)$ and $(1,1)$ provided that $\vec{F} = x^2\hat{\imath} + y^2\hat{\jmath}$. [4 *Marks*]

(b) Verify the Green's theorem for $\int_C \{(xy + y^2)dx + x^2dy\}$

where C is bounded by $y = x$ and $y = x^2$. [4 *Marks*]

(c) Show that $\iiint_V \frac{dv}{r^2} = \iint_S \frac{\vec{r} \cdot \hat{n}}{r^2} ds$. [4 *Marks*]

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